Introducing Trigonometry without the Jargon

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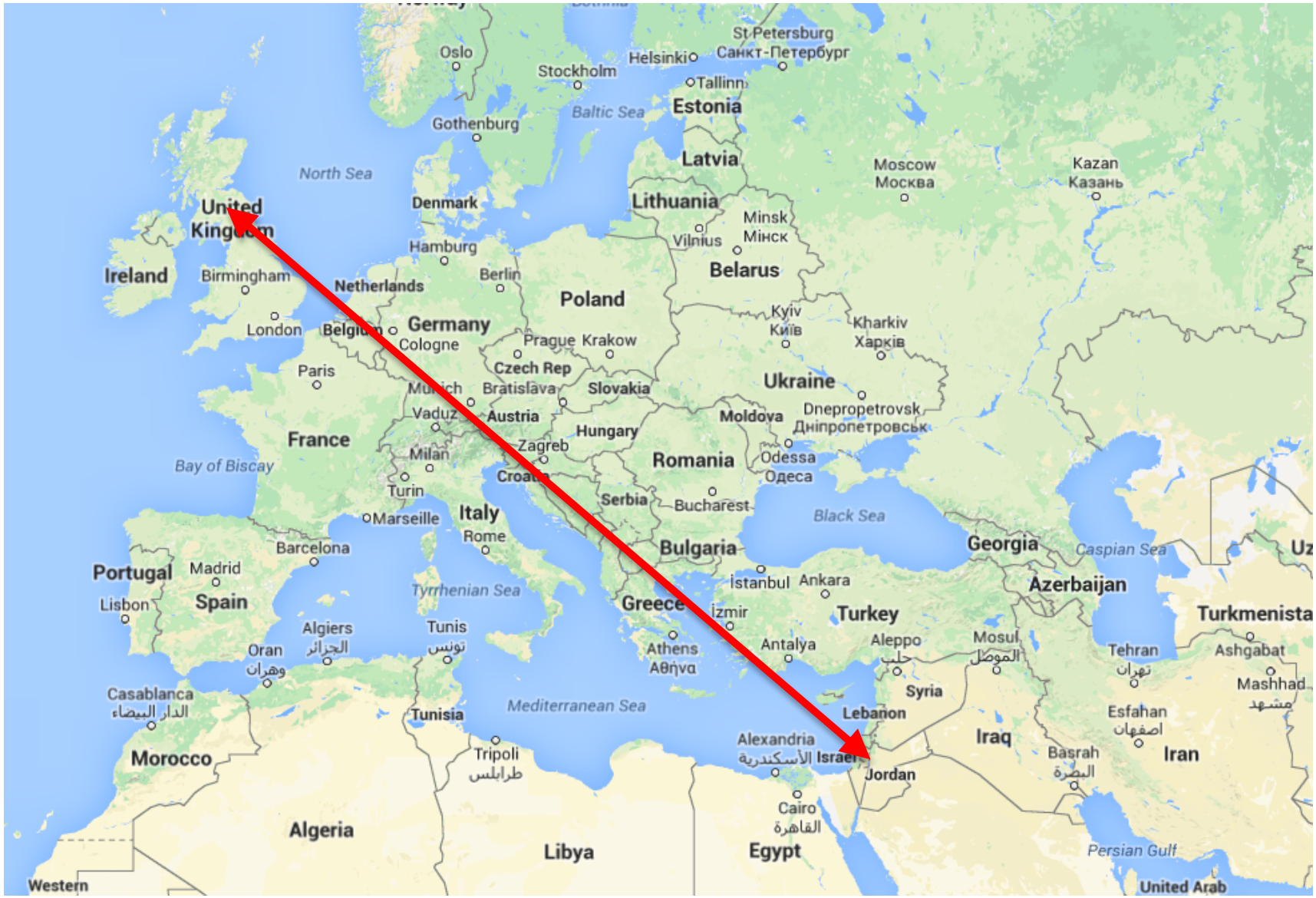
www.calculatorsoftware.co.uk/nspire

Journey: 2496 miles (4015km)



This talk will have a….

A Beginning

The challenges … and the challenge!

A Middle

The route that I took.

An End

The benefits

And you can download all that you see today from

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Why do students find Trigonometry hard?

Talk to your neighbour about why you think students find trigonometry hard.

Why do students find Trigonometry hard?

New words: opposite, adjacent, hypotenuse
sine, cosine, tangent
a ratio that's not written like "a:b"

New ideas: functions that are not just "f(x)"
inverse functions
acceptable domains of functions

New notation: sin(x), cos(x), tan(x)

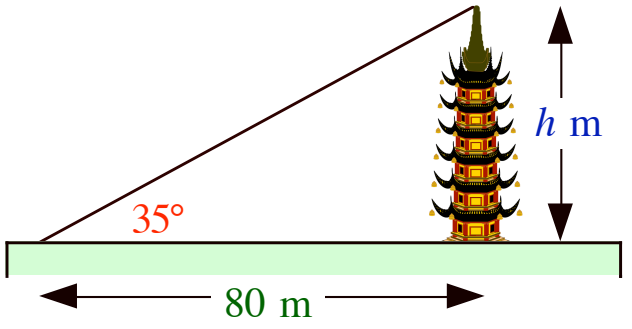
Need to know: Similar Triangles

The Challenge

Setting the Scene

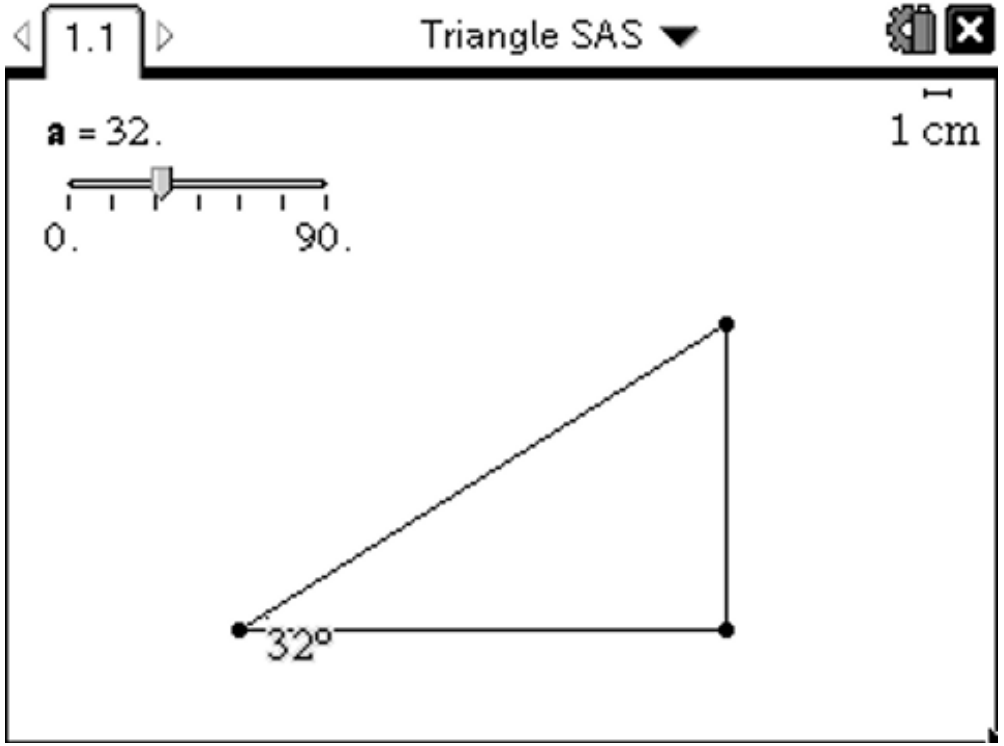
From a point 80 metres away from the foot of the pagoda, the angle of elevation of the top is measured as 35°.

Calculate the height of the pagoda.



An Interactive Geometry Construction

Triangle SAS.tns



Name lengths.
Evaluate a calculation.
Students then played.

Student Contributions

Teacher: “Have you noticed anything that you want to talk about?”

Students: 9 said Yes, 13 said No.

“Why when you put it up to 90, does it go all flat?”

“When it’s 45 degrees, it’s exactly 1”

“When you keep the angle the same, and you change the length of the sides, the ratio is the same”

Teacher: “Who else noticed this, but didn’t think it was important?

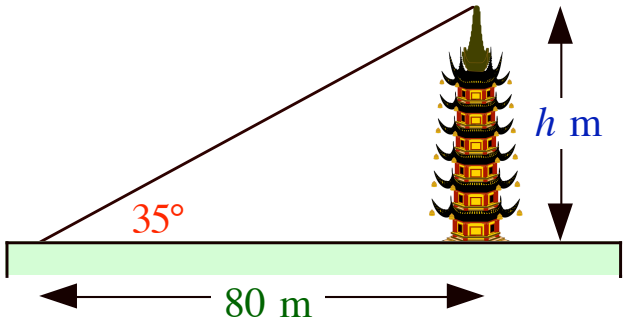
The **majority** of the class considered that the division answer remaining unchanged when you change the size of the triangle was **not important.**

This is the whole **POINT** of trigonometry!

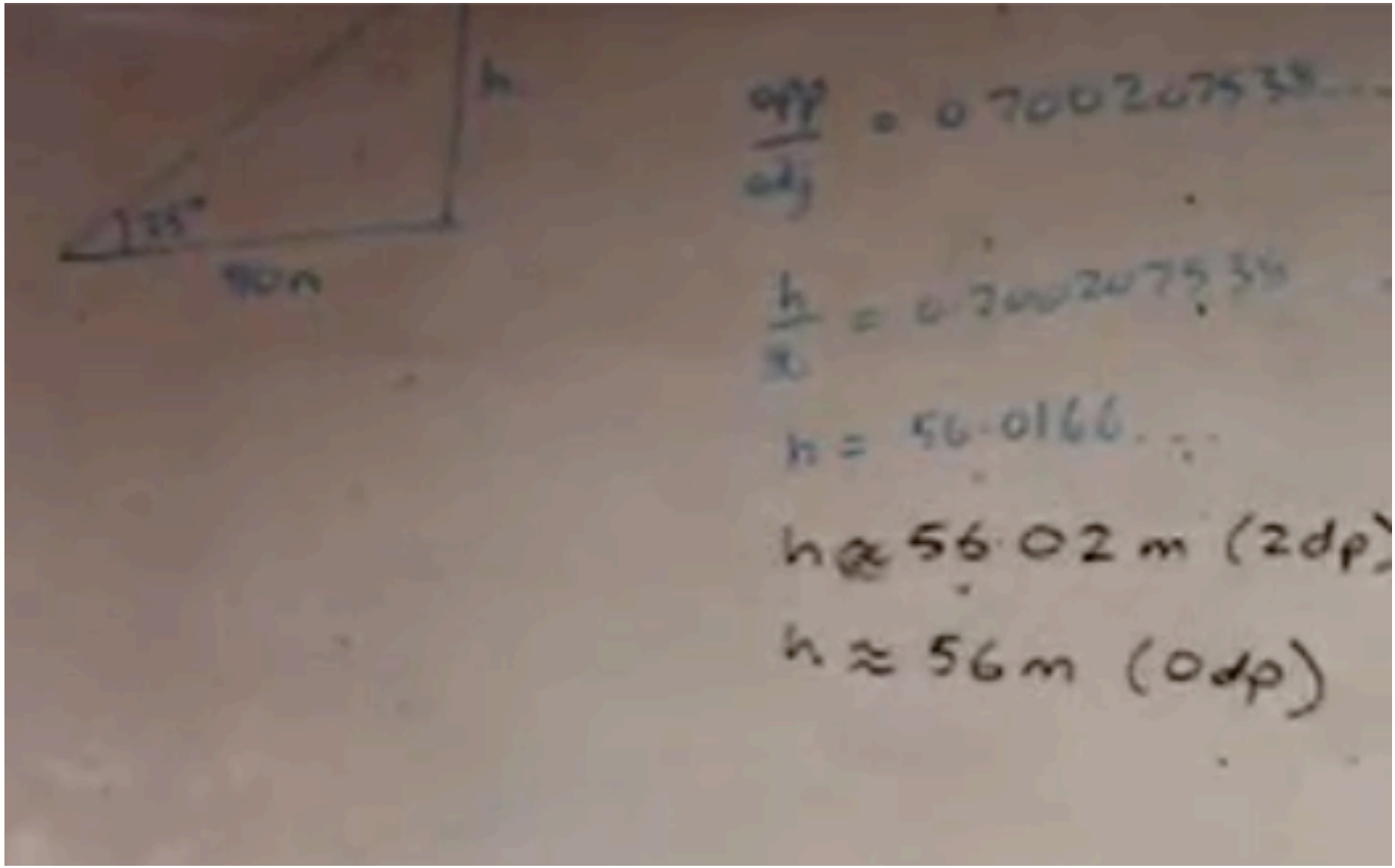
Back to the Question

From a point 80 metres away from the foot of the pagoda, the angle of elevation of the top is measured as 35°.

Calculate the height of the pagoda.



Students tried to recreate dimensions on handheld



$\frac{opp}{adj} = 0.700207538\ldots$

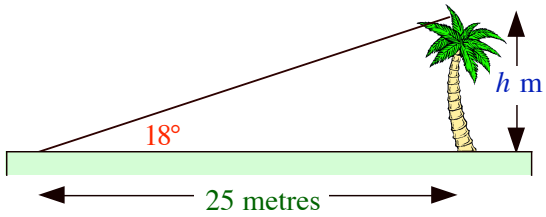
$\frac{h}{90} = 0.700207538\ldots$

$h = 56.0166\ldots$

$h \approx 56.02\text{ m}$ (2dp)

$h \approx 56\text{ m}$ (0dp)

Four Similar Questions

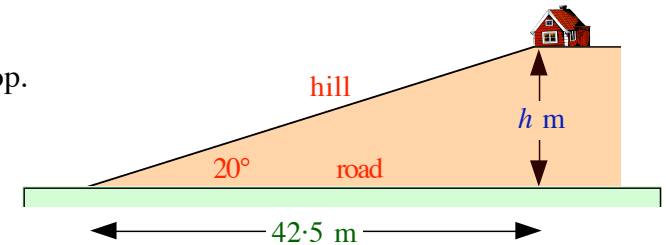
4. 

The angle of elevation of the top of a tree from a point 25 metres from its foot is 18°.

Calculate the height of the tree.

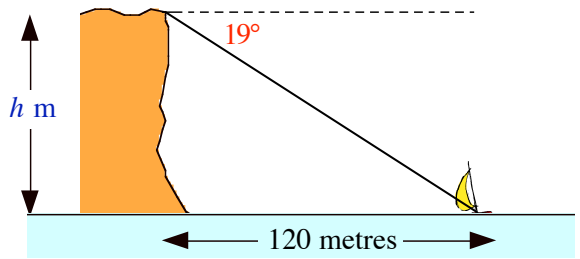
5. A hill runs up from a main road to the house at the top. The hill makes an angle of 20° to the road.

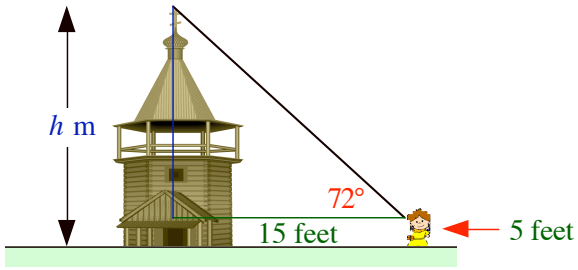
Calculate how high the house is above the road.



7. From the top of a cliff, a small boat is observed at an angle of depression of 19°.

If the boat is 120 metres from the foot of the cliff, find the height of the cliff.



8. 

A girl, who's eyes are 5 feet above ground-level, is attempting to measure the height of this tower.

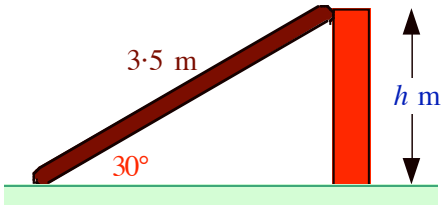
She is standing 15 feet from the tower looking to the top at an angle of 72° to the horizontal.

How high is the tower ?

Source: TeeJay Publishers 'Intermediate-2-Credit Book 1' page 122

Four More Questions Using New Sides

4.



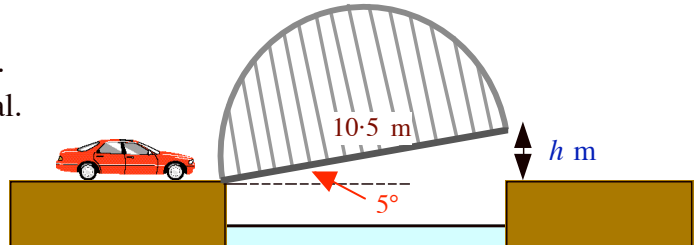
A plank is 3·5 metres long, and lies at an angle of 30° to the ground.

It is just touching the top of a wall.

Calculate the height (h metres) of this wall.

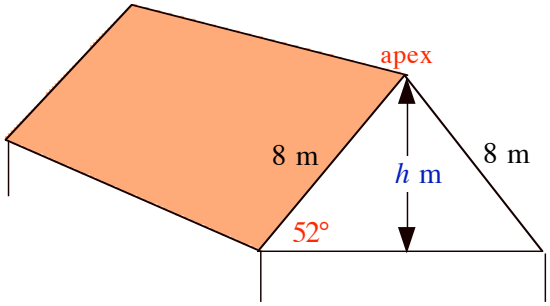
5. A bridge across a shallow river is 10·5 metres long. It is shown making an angle of 5° to the horizontal.

How much higher is the bridge at one end than it is at the other at this stage ?



7. The angle of slope of a roof is 52°.

If the sloping part is 8 metres long, how high is the apex above the foot of the roof ?



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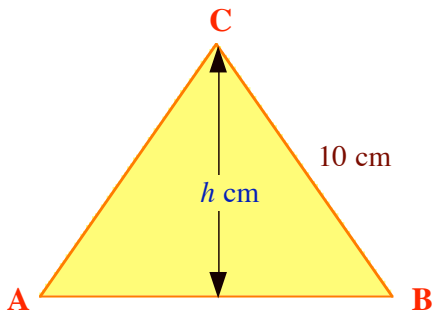


Triangle ABC is an **equilateral** triangle of side 10 cm.

(a) Write down the size of BAC.

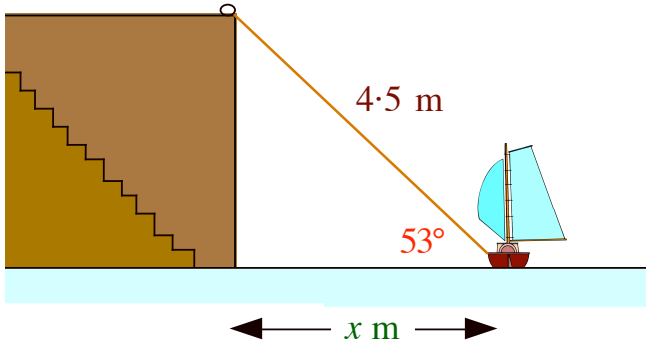
(b) Calculate its height (h cm), **using trigonometry**.

(c) Now check your answer using **Pythagoras' Theorem**.

Source: TeeJay Publishers 'Intermediate-2-Credit Book 1' page 126

Two Final Questions Using New Sides

5.

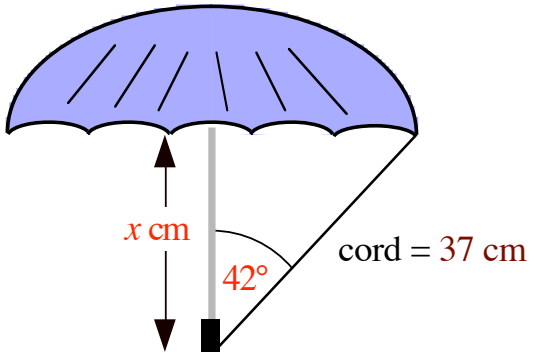


A yacht is moored to the quay wall by a rope 4·5 metres long. When the rope is taut, it makes an angle of 53° with the surface of the sea.

How far is the yacht from the quay wall ?

6. This umbrella has a cord joining the end of the handle to one of the "prongs" of the cover.

Calculate the length of the handle shown (x).



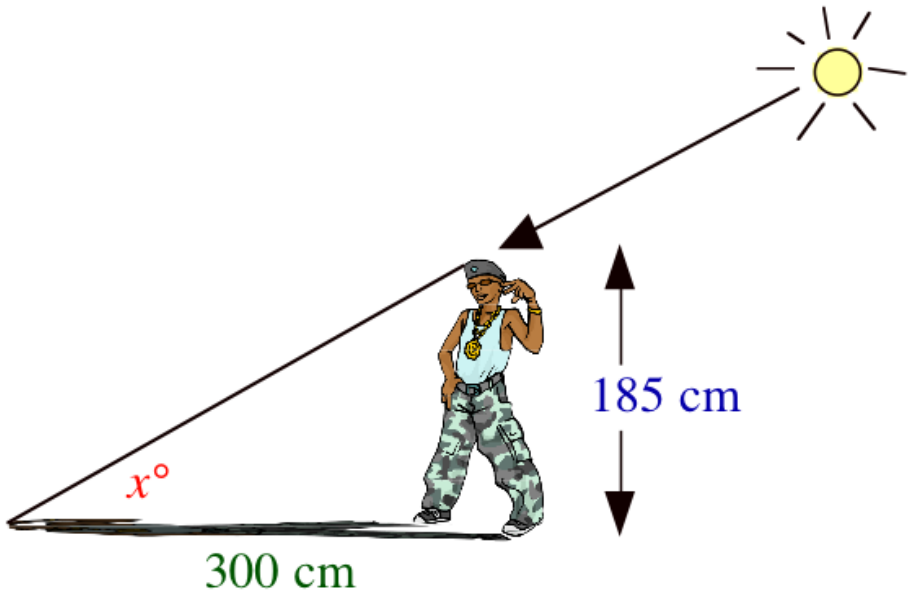
R ∧

Source: TeeJay Publishers 'Intermediate-2-Credit Book 1' page 129

Now 'Backwards'

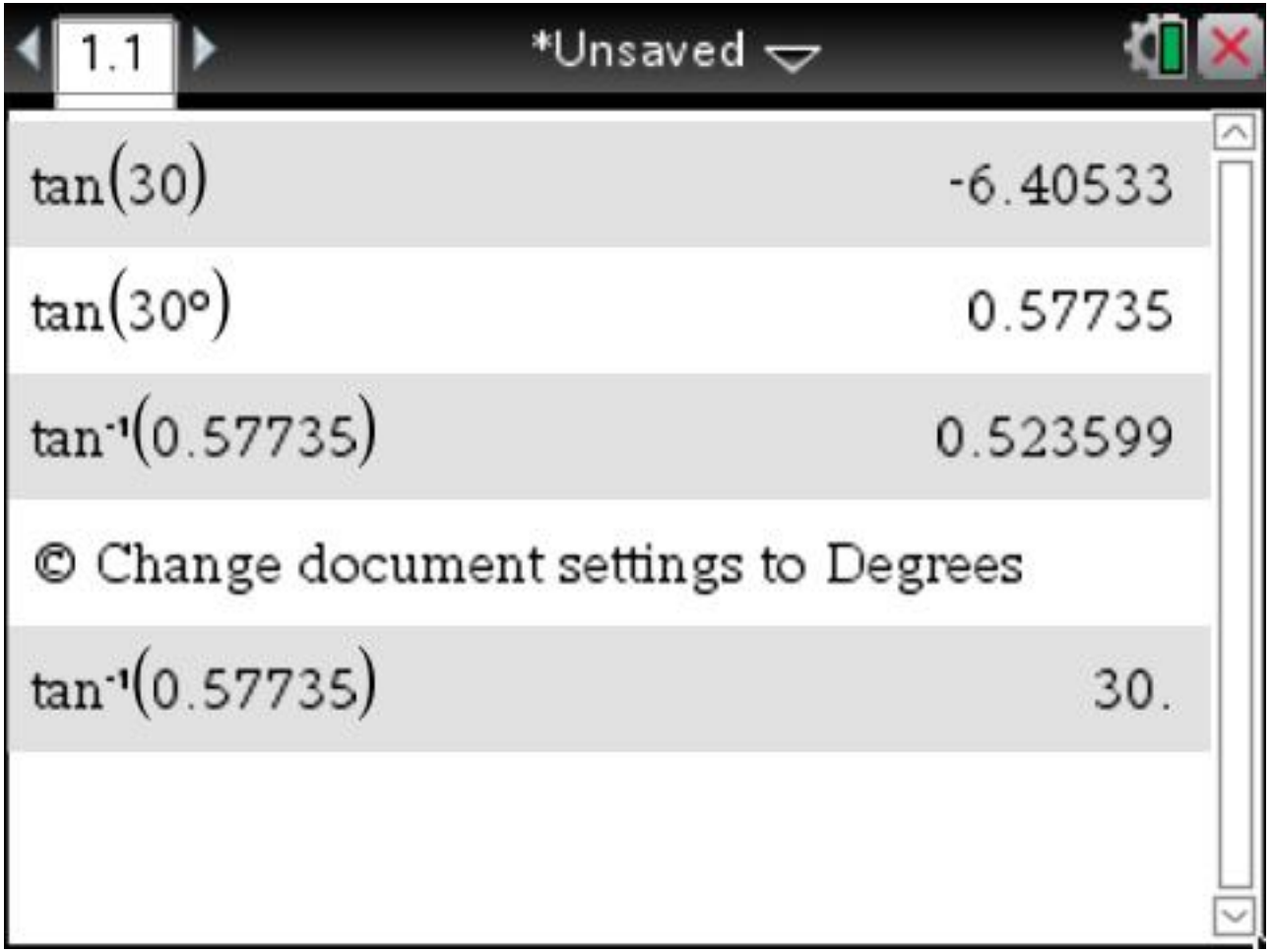
5. Winston is 185 centimetres tall. In the sunshine he casts a shadow on the ground 300 centimetres long.

   Find the angle of elevation ($x°$) of the sun.



Source: TeeJay Publishers 'Intermediate-2-Credit Book 1' page 124

Trigonometric Function Names, Radians and Degrees



Formal Definitions

2.1 | 3.1 | 4.1 | Triangle SAS

The ratio $\frac{opp}{hyp}$ is called the **sine** ratio

The ratio $\frac{adj}{hyp}$ is called the **cosine** ratio

The ratio $\frac{opp}{adj}$ is called the **tangent** ratio

"**S**^{O}H **C**^{A}H **T**^{O}A"

3.1 | 4.1 | 4.2 | Triangle SAS

We write **sin** for **sine**

We write **cos** for **cosine**

We write **tan** for **tangent**

It just saves a bit of space. Nothing more.

4.1 | 4.2 | 4.3 | Triangle SAS

When you **sine** an angle, you get the value of the ratio:

sin(angle)=ratio

When you **inverse sine** the value of the ratio, you get the angle.

angle=**sin**$^{-1}$(ratio)

4.2 | 4.3 | 4.4 | Triangle SAS

Similarly, **cos** and **inverse cos**:

cos(angle)=ratio

angle=**cos**$^{-1}$(ratio)

Similarly, **tan** and **inverse tan**:

tan(angle)=ratio

angle=**tan**$^{-1}$(ratio)

4.3 | 4.4 | 4.5 | Triangle SAS

Other trigonometric functions that you will meet in the future:

$$\csc(x)=cosec(x)=cosecant(x)=\frac{hyp}{opp}=\frac{1}{\sin(x)}$$

$$\sec(x)=secant(x)=\frac{hyp}{adj}=\frac{1}{\cos(x)}$$

$$\cot(x)=cotangent(x)=\frac{adj}{opp}=\frac{1}{\tan(x)}$$

4.4 | 4.5 | 4.6 | Triangle SAS

And finally, all that the **trigonometry functions** do is convert from angles and ratios, to save us drawing similar triangles.

The **inverse trigonometric functions** allow us to swiftly work out angles from knowing the ratio of sides.

Benefits

- the decimals were generated in front of them by a **known process**, based on a diagram

- used all the ratios in context, **before** formally defining them

- notation was only introduced **as and when** it was needed

- the topic did not have to be re-taught later on!

Anyone can get it wrong quickly.

Take your time.

Get it right...

...first time.

Will You Try it?

Want Copies of Everything?
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Thank you for coming to my talk.

Nevil Hopley

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